Footnote (1) from the article titled "On the Significance and Needs of Mathematical Logic," by Jan Łukasiewicz, published in Polish in *Nauka Polska*, No. 10, pp. 604-620, 1929.

Since these divagations would be certainly non-understandable to the reader not knowing mathematical logic, therefore I will try, in the most straightforward way, to sketch the theory I talk about in the text.

The combination of letters marked by ordered numbers below may be treated, for the time being, as figures in some game, not having any meaning. The following three figures are assumed beforehand as belonging to this game:

(1) *CCpqCCqrCpr*

(2) *CCNppp*

(3) *CpCNpq*

The game relies on creating new figures based on figures already belonging to the game, by using the following two rules:

I. *The Rule of Substitution*. From the game figure *F*, one can create a new figure, by substituting, in a way that one removes any lower-case letter from figure *F* and all letters identical with it, and fills thus obtained gaps identically, either (a) with a single lower-case letter different from the one removed, or (b) with two letters, of which the first one is "*N*" and the second is any lower-case letter, or (c) with three letters, of which the first one is "*C*" and the next two are any lower-case letters.

II. *The Rule of Detachment*. From figure *F*, one can create a new figure by detachment, then and only then, if figure F begins with the letter "*C*" and its certain part, directly following this "*C*", is a figure of this game; the end part of figure "*F*", remaining after the detachment from "*F*" the beginning "*C*" and a figure of the game following this "*C*", becomes a new figure of the game.

These rules will be clear, if we look at the following examples of new figures created according to the rules:

(4)	*CCpCsqCCCsqrCpr*	1 *q/Csq*
(5)	*CCpCNpqCCCNpqrCpr*	4 *s/Np*
(6)	*CCCNpqrCpr*	5, 3

(7) *CCCNpprCpr* 6 *q/p*

(8) *CCCNpppCpp* 7 *r/p*

(9) *Cpp* 8, 2

Figure no. 4 is created from the first one, by substituting in it the letter "*q*" by three letters "*Csq*", according to point (c) from the first rule. Figure no. 5 is created from the fourth one by substituting in it the letter "*s*" by two letters "*Np*", according to point (b) of the first rule. Figure no. 7 has been created from figure no. 6, by substituting in it the letter "*q*" by the letter of "*p*", according to point (a) of the first rule. In a similar way, figure no. 8 was created from that of no. 7. Figure no. 6 was created from figure no. 5 by detaching from it both the beginning "*C*" and directly following it figure no. 3, "*CpCNpq*", according to the second rule. In a similar way, figure no. 9 was created from figure no. 8, by detaching from it the beginning "*C*" and directly following it figure no. 2, "*CCNppp*", according to the second rule.

I expect that, after studying these examples, the reader will begin understanding the game, although without knowing the meaning of these figures. Their meaning is explained in the following.

Lower-case letters are predicate variables, that is, variables whose values can only be sentences. The expression "*Np*", which reads "not true that *p*", is a negation of the sentence "*p*". Thus, if "*p*" means "today is Friday", then "*Np*" means "it is not true that today is Friday". The expression "*Cpq*", which reads "if *p*, then *q*", is an implication with an antecedent "*p*" and a consequent "*q*". Thus, if "*p*" means "today is Friday" and "*q*" means "tomorrow is Saturday", then "*Cpq*" means: "if today is Friday, then tomorrow is Saturday".

Figures 1-3 are axioms of the theory of deduction. One can read them as follows:

(1) if (if *p*, then *q*), then [if (if *p*, then *q*) then (if *p*, then *q*)]

(2) if (if not true that *p*, then *p*), the *p*

(3) if *p*, then (if not true that *p*, then *q*)

The first axiom is the law of conditional syllogism and was known already by Aristotle. The second axiom is a certain form of reasoning called "a reduction to absurd", and was used as early as in proofs by Euclid. The third axiom expresses an idea known already to Duns Scotus, who claimed that from two contradictory sentences, for example

"Socrates is" and "Socrates is not", there follows any sentence, for example, "a stick is in the corner". Whoever is going to thought these axioms out, he will undoubtedly accept their truthfulness.

The rule of substitution corresponds to the principle of deductive reasoning, known already in Aristotle's syllogistic, and named in traditional logic "*dictum de omni*". This principle states that whatever holds for all subjects of a certain species, it also holds for certain subjects of this species. Axioms 1-3 and all theorems derived from them hold for all sentences "*p*", "*q*", "*r*", "*s*", and therefore also hold for negations and implications, which are substituted in place of these sentences.

The rule of detachment corresponds to the principle of deductive reasoning, known already in Stoic dialectics, and named in the traditional logic "*modus ponens*". This principle states that, if the implication is valid, for example, such as this one "*CCCNpppCpp*", and valid is the antecedent in it, "*CCNppp*" in our example, then we have the right to consider its consequent valid. Both rules are obvious.

Thanks to these rules, one can prove an unlimited number of theorems, and among them the most important laws of reasoning. Thus, theorem (9) proved above, is the law of identity for sentences and states that "if *p*, then *p*".

Proves of all theorems are complete, which means that they do not have any gaps, and are formalized, which means that one can verify them based exclusively on the appearance of these theorems, disregarding their meaning.

At the end, let me add a few historical remarks. The notation used in this sketch and the arrangement of axioms originated from me. Mine is also the way of writing proofs. The idea of formulating the rules of substitution in its simplest form, as presented above, is thanks to Dr. Tarski. The founder of the modern theory of deduction is Frege (*Begriffsschrift*, 1879), who was also the first one who understood the difference between the thesis of the system, that is, an axiom or a theorem, and the rule of inference, such as the rule of detachment, for example. Frege was also the first one who cared about the completeness of proofs; completeness of proofs is also especially emphasized in logical works of professors Sleszyński and Zaremba.

Translated by J. Zalewski