INTEGRATING

WITH THE

CORADI INTEGRAPH

INVENTED BY ABDANK-ABAKANOWICZ

PRACTICAL TECHNICAL PROBLEMS

WITH EXAMPLES OF SOLUTIONS

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PREFACE

In practice it is often necessary to evaluate the integrals of any given curves, and then a knowledge of the integral curve is desired. Generally this work entails much trouble, particularly if the curves are only given graphically. In this case the integraph can be used to advantage.

The purpose of this brochure is to describe the manipulating and the possibilities of adopting the integraph for technical work and to give accurate indication as to how the integral curves are evaluated, what influence the different positions of the integraph have on the curves, and how these positions are to be taken into consideration when evaluating the results.

This task falls naturally into three parts. In the first part the fundamental principles of the instrument, and also its construction and method of handling are described. The second part deals with the most important properties of the different integral curves, and the last part contains examples of application.

The possibilities of application of the integraph are far from being exhausted by the examples given in this brochure. It should, however, be possible to solve current technical problems with the integraph by following the examples given here. Integration can be performed much more quickly with the integraph than by graphical or numerical methods. For this reason it is to be hoped that this brochure will cause the integraph to be more frequently used for integrating than it has been in the past.

Here I would not omit to thank Mr. G. Coradi for the advice he has given; he has also, as an expert, undertaken the description of the apparatus.

THE AUTHOR.

I. THE INTEGRAPH

1. Fundamental mathematical principles

a) The indefinite integral

Let $y = g(x)$ be a function of which we wish to consider the indefinite integral

$$\int g(x)\,dx = \int y\,dx\,.$$

This integral is a new function, $Y = G(x)$, which has the property that its differential quotient $\frac{dY}{dx}$ is proportional to the function $y = g(x)$; i.e.:

$$\frac{dY}{dx} = \frac{g(x)}{\lambda}. \qquad \text{Definition of the indefinite integral}$$

Here λ is a quantity independent of x; it has the same dimensions as x and we shall call it the *base* of the integration. Base

In mathematics the base is generally adopted as a unit of length and is therefore in most formulae not written at all. In what follows, however, it plays an important part.

Both the function $y = g(x)$ and also the function $Y = G(x)$ can each be represented by a curve; these we call the fundamental curve and the integral curve respectively. Fundamental curve

The indefinite integral and thereby the integral curve, is not completely determined by the definition given above, for every curve which is produced from Y by parallel movement in the direction of the Y axis, possesses the property mentioned. The integral curve is only definitely determined when it is stated how great its ordinate is to be for $x = x_0$. Integral curve

b) Construction of the integral curve

The integral curve can be constructed from its definition. From the point M (Fig. 1) the length λ (base length of the integration) $M\bar{M}$ is drawn to the left. The ordinate of $\bar{M}$ cuts the x axis at U. Then UM has the same direction as the tangent to the sought integral curve, since $\frac{dY}{dx} = \tan\tau$.

If the problem is to be solved mechanically, we have only to move a pencil or a drawing board in the given direction UM. The problem can be solved,

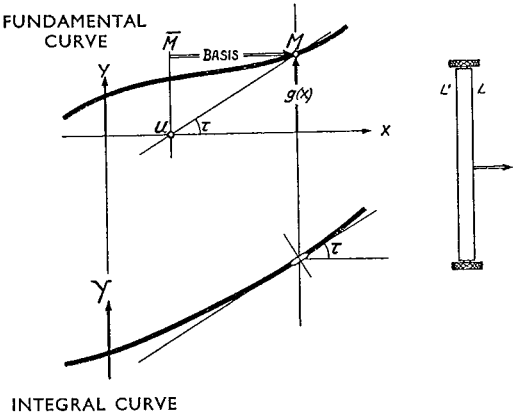


Fig. 1

INTEGRAL CURVE AND FUNDAMENTAL CURVE

as Abdank-Abakanowicz perceived, with an *integrating wheel.* This is nothing else than a small sharp-edged wheel which can move forward on the paper only in the direction of its plane of rotation (the plane normal to the axis of rotation). If the axis of the integrating wheel is always guided at right angles to the direction UM and moved forward uniformly with the ordinate of M, the integrating roller describes exactly the integral curve of the fundamental curve in question. The construction of the Coradi integraph consists in a small wheel being guided in the manner just described.

2. The construction of the integraph

a) Fundamental principles

Fig. 2a and the perspective pictures show the construction of the integraph. It consists of two bars L and L' which remain on the wheels r always parallel to the y-axis and are only movable in the direction of the x axis. On these bars two carriages W and W' can move freely in the direction of the x axis. One carriage bears, on an arm, the tracer t and the other carriage the integrating wheel i and along with this the pencil k. From the point M a rod projects, which is supported at U so that it is pivotable and movable, and carries, at the other end from U, a carriage W''. This rod is called the *directrice bar.* With the help of the parallel linkage, the carriage W'' causes the integrating wheel to move in the direction of the rod MU.

Directrice bar

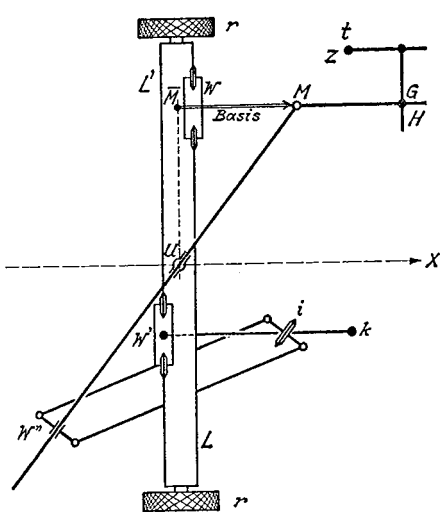


Fig. 2a

SKETCH MODEL OF THE INTEGRAPH

b) Detailed description

The integraph rests on the drawing board at three points, two of which are formed by the wheels r and r' fixed to the axle O (*Fig. 2b'*) and the third by the tracer t or its support. The two wheels r and r' fixed on a common axle guide the apparatus in a direction at right angles to this axis, so that the apparatus may be moved on the drawing board in this direction on a straight line of any desired length. The U-shaped guide rail, in which the wheels rr rest on ball bearings between two stirrups, has two straight guide pieces L and L', parallel to each other, which are directed at right angles to the direction of movement of the apparatus and for their part parallel to the axle O of the wheels rr.

Guiding carriage

In the channel of the front guide piece L, the guiding carriage W moves between three rollers; this carriage carries the base bar B which is graduated. At its end is the tracer t, supported in a movable sleeve. On the base bar B is also a sleeve provided with vernier and micrometer gear, which serves to set the desired base length. This sleeve carries the movable vertical axle M, in whose upper part the straight edge D is firmly clamped. This straight

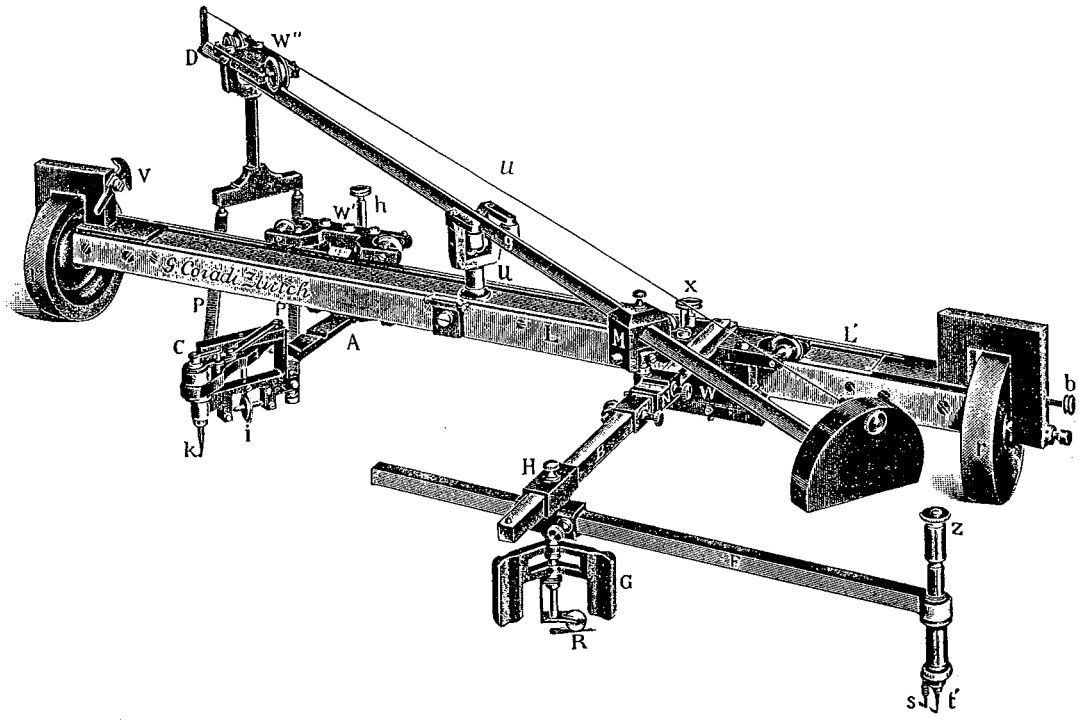


Fig. 2b

Integrating carriage

edge is adjustable lengthways. In the rear straight guide L' the integrating carriage W' moves, on which the integrating wheel i and the perforated pencil socket are fitted. Into this socket a pencil holder or a drawing pen can be inserted. By means of a small parallelogram the drawing pen is set automatically into the line to be drawn. In the middle of the guide rail a second vertical axle U is provided. It carries a frame g with an easily rotatable roller, the straight edge D sliding in the groove of this roller. On the straight edge the carriage W'' moves, which is connected by the parallelogram pp to the frame C of the integrating wheel. In this way the plane of rotation of the integrating wheel is always guided parallel to the straight edge.

When the guiding arm is set so that the straight edge stands at right angles to the guide rail — in other words, when the straight edge is in the direction of the x axis — the plane

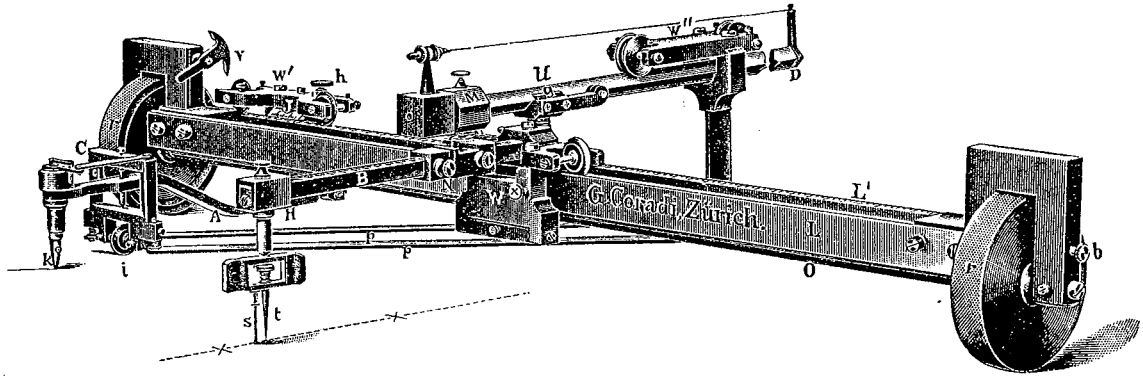


Fig. 2b'

of rotation of the integrating wheel is also set parallel to the x axis (*Fig. 2c*). We call this the orthogonal position of the integraph. In this position the distance between the two axes of rotation of the straight edge is equal to the base of the apparatus. (The base corresponds to the moving arm of the polar planimeter.)

Orthogonal position

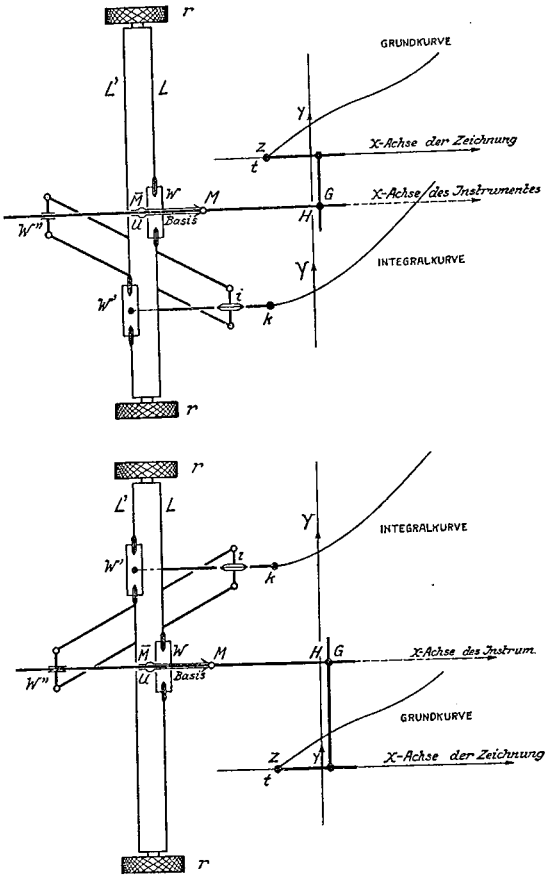
Base

When the apparatus moves in the orthogonal position, there is no lateral movement of the integrating carriage. In this position the guide carriage can be fixed by means of a free-falling pin, indicated with X. As soon as the guide carriage has reached the orthogonal position, this pin engages in a transverse groove which is located in a steel part, laterally displaceable by means of screws. The pin X can be secured by a quarter turn when not in use. It then remains suspended.

Stop pin X

The front rotating axis of the straight edge is also supported in balls and is located in the sleeve N, which is displaceable on the base bar B. The base bar is graduated in mm and in $^1/_{10}$ inch. With the help of the micrometer and the vernier, the sleeve N can easily be adjusted accurately to $^1/_{10}$ mm or to $^1/_{100}$ inch. The table pasted in the box containing the instrument gives the vernier adjustments which correspond to the whole numbers of the base length.

Setting the base



Grundkurve = Fundamental curve
X-Achse der Zeichnung = X-axis of the drawing
X-Achse des Instrumentes = X-axis of the instrument
Integralkurve = Integral curve

Fig. 2 c

INTEGRAPH IN ORTHOGONAL POSITION

3. Handling the integraph

If an integral curve is to be constructed with the help of the integraph, the apparatus must first be directed to accord with the axes of the given drawing. Attention must then be paid to the following facts:

Orientation of the apparatus

1. The moving of the whole apparatus must take place in the direction of the x axis.
2. In the orthogonal position the tracer pin must move exactly on the x axis of the drawing.

In order to obtain this, proceed as follows: Fix the guide carriage in the orthogonal position by means of the pin X, then take hold of the apparatus at the lefthand and righthand in the wheel stirrups and lift it a little from the paper. Then move the apparatus until the tracer comes on to the x axis of the drawing, the base and the straight edge lie at the same time parallel to the x axis.

In order to correct the position when setting the directions, the guide rail L or L' may be used as sighter on an ordinate, since guide rail and ordinate must also be parallel to each other.

If the apparatus is moved in the x direction with the zero position fixed, the tracer pin should remain on the chosen abscissa axis. Any deviations are to be corrected by making a further change in the whole position of the apparatus.

Device for moving the pencil

The apparatus can be more easily set in direction, if an appliance is used which allows the tracer pin to move laterally (cf. perspective *Fig. 2b*). This appliance consists of a special sleeve H, in which the ruler F can be firmly clamped by pressure screws. This ruler can be displaced by about 25 cm or 10 inches and carries at its end the drawing socket Z. For integraphs which are provided with this appliance, a roller R, rotatable in all directions, serves as third supporting point. Beside this roller there is the guide handle G. The tracer t fitted on the displaceable bar, swings quite freely and should never be used for guiding the apparatus.

Guiding handle G

When the apparatus has been set, this appliance also allows several integrations to be carried out without having to re-set the apparatus in direction. It is only necessary to set the tracer on to the new x axis.

Advantageous starting place

Before tracing round a figure, the integrating carriage W' must always be set so that its track is as free as possible. Then attention has to be paid to the following fact: If the integraph is moved in the direction of the base, the integrating carriage W' tends to run towards the end of that half of LL' on which the guide carriage W is located. If the apparatus is moved in the opposite direction, the integrating carriage moves towards the other end. The carriage moves all the quicker, the further the guide carriage is distant from the middle. Taking this as a rule, always consider first of all where the integrating carriage tends to run to and then set it down on to the drawing as close as possible to the opposite end. The screw h provided on the integrating carriage W', serves for raising and lowering the integrating wheel. The wheel may also be raised and displaced by hand.

The ordinate differences of the integral curve may also be read on the integraph accurate to one tenth of a millimetre. The ruler L' has a mm-graduation and the carriage W' is provided with a vernier.

With a short base (for instance 2″ to 3″), the lateral movement of the tracer carriage W from the orthogonal position in the x direction, positive or negative, is limited. This is both because of the maximum permissible angular movement of the integrating wheel frame C, and also because of the position of the carriage W'' on the guide rail L'. The carriage W'' may in fact touch the guide rail. Large figures can therefore not be traced round with a small base.

The tracer support s causes the tracer pin to swing somewhat above the plane of the drawing. In order to mark the beginning of the tracing, prick the tracer pin into the paper by pressing it slightly with the finger; a spring raises the pin again from the paper.

Displaced ordinates

The tracer pin, or its arm, is firmly clamped in a sleeve H on the base bar. The tracer pin can be moved with the sleeve in such a way that the abscissa difference between tracer pin and pencil is a round figure. The common ordinates of the base and of the integral curves are then displaced with respect to each other by this distance. With small bases it is possible to set the tracer pin on the ordinate of the pencil. (Abscissa difference 0, corresponding ordinates.) In this manner, however, the path through which the integrating and the guide carriages can be moved is reduced, since in this case they cannot pass beside each other. For the sake of clearness, our drawings are all made with corresponding ordinates.

Base direction

For evaluating the drawing it is also important to know what direction the base of the integraph had when the drawing was made. As direction of the base we designate the direction of the distance $\bar{M}M$ from $\bar{M}$ to M (cf. *Fig. 3*). If the base has the same direction as the positive x axis (guide carriage W on the same side as the positive x) the positive Y direction of the integraph curve is directed the same as the positive y axis of the fundamental curve. If, however, the base direction and the x direction are contrary to each other, the positive

directions of the ordinates change at each integration (cf. *Fig. 3, 4b, 5a, 6b, 7, 8*). It is preferable on each drawing to indicate diagrammatically the position of the instrument, and also the length and direction of the base.

All the illustrations in this brochure that represent integral curves are reduced pictures of curves made with the Coradi integraph type II. The given dimensions refer to the reduced pictures and can directly be measured there. The base lengths are also reduced. In reality the smallest possible base with this type of integraph is 5 cm.

The present essay being a translation from German the evaluation of the figures was throughout done in cm. Any figure or formula may as well be applied for inches, square inches etc.

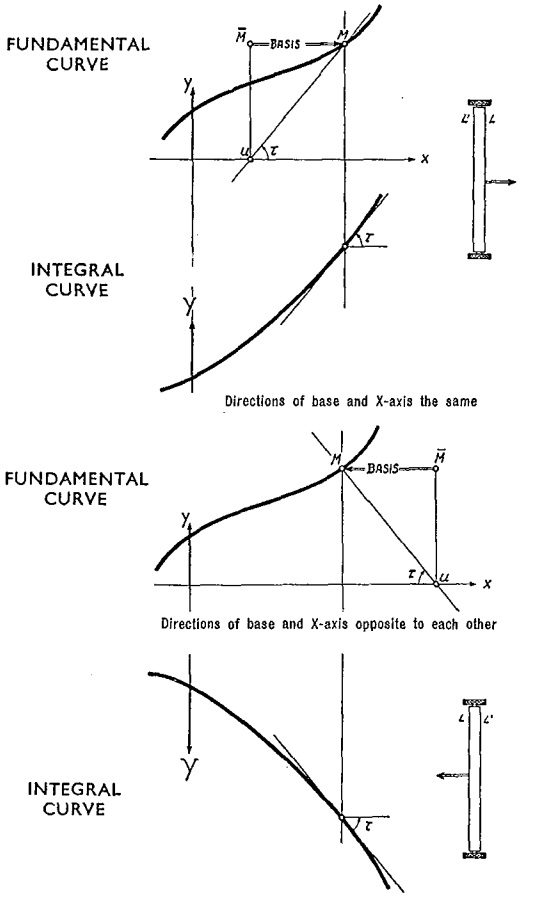


Fig. 3

INFLUENCE OF THE DIRECTION OF THE BASE