# Rough Sets

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Rough set theory, introduced by Zdzislaw Pawlak in the early 1980s [11, 12], is a new mathematical tool to deal with vagueness and uncertainty. This approach seems to be of fundamental importance to artificial intelligence (AI) and cognitive sciences, especially in the areas of machine learning, knowledge acquisition, decision analysis, knowledge discovery from databases, expert systems, decision support systems, inductive reasoning, and pattern recognition.

The rough set concept overlaps—to some extent —with many other mathematical tools developed to deal with vagueness and uncertainty, in particular with the Dempster-Shafer theory of evidence [15]. The main difference is that the Dempster-Shafer theory uses belief functions as a main tool, while rough set theory makes use of sets—lower and upper approximations. Another relationship exists between fuzzy set theory and rough set theory [13]. Rough set theory does not compete with fuzzy set theory, with which it is frequently contrasted, but rather complements it [1]. In any case, rough set theory and fuzzy set theory are independent approaches to imperfect knowledge. Furthermore, some relationship exists between rough set theory and discriminant analysis [7], Boolean reasoning methods [16], and decision analysis [14].

One of the main advantages of rough set theory is that it does not need any preliminary or additional information about data, such as probability distribution in statistics, basic probability assignment in the Dempster-Shafer theory, or grade of membership or the value of possibility in fuzzy set theory [2].

**Although the burgeoning methodology of approximation sets has been successful in many real-life applications, there are still several theoretical problems to solve.**

### Basic Concepts

In this article we will assume that the information about the real world is given in the form of an *information table* (sometimes called a decision table). Thus, the information table represents input data, gathered from any domain, such as medicine, finance, or the military. An example of such an information table is given in Table 1.

Rows of a table, labeled e1, e2, e3, e4, e5, and e6 in Table 1, are called *examples* (objects, entities). Properties of examples are perceived through assigning values to some variables. We will distinguish between two kinds of variables: *attributes* (sometimes called condition attributes) and *decisions* (sometimes called decision attributes). Usually a single decision is all that is required. For example, if the information table describes a hospital, the examples may be patients; the attributes, symptoms and tests; and the decisions, diseases. Each patient is characterized by the results of tests and symptoms and is classified by the physicians (experts) as being on some level of disease severity. If the information table describes an industrial process, the examples may represent samples of a process taken at some specific moments in time; attributes, the parameters of the process; and decisions, actions taken by the operators (experts).

**Table 1.** Information Table.

| | **Attributes** | | | **Decision** |
|---|---|---|---|---|
| | Headache | Muscle_pain | Temperature | Flu |
| e1 | yes | yes | normal | no |
| e2 | yes | yes | high | yes |
| e3 | yes | yes | very_high | yes |
| e4 | no | yes | normal | no |
| e5 | no | no | high | no |
| e6 | no | yes | very_high | yes |

The main concept of rough set theory is an *indiscernibility relation*, normally associated with a set of attributes–for example, the set consisting of attributes *Headache* and *Muscle_pain* from Table 1. Examples e1 and e2 are characterized by the same values of both attributes: for the attribute *Headache* the value is *yes* for e1 and e2 and for the attribute *Muscle_pain* the value is *yes* for both e1 and e2. Moreover, example e3 is indiscernible from e1 and e2. Examples e4 and e6 are also indiscernible from each other. Obviously, the indiscernibility relation is an equivalence relation. Sets that are indiscernible are called *elementary sets.* Thus, the set of attributes *Headache* and *Muscle_pain* defines the following elementary sets: {e1, e2, e3}, {e4, e6}, and {e5}. Any finite union of elementary sets is called a *definable set.* In our case, set {e1, e2, e3, e5} is definable by the attributes *Headache* and *Muscle_pain,* since we may define this set by saying that any member of it is characterized by the attribute *Headache* equal to yes and the attribute *Muscle_pain* equal to yes or by the attribute *Headache* equal to no and the attribute *Muscle_pain* equal to no.

Due to the concept of indiscernibility relation, it is very simple to define redundant (or dispensable) attributes. If a set of attributes and its superset define the same indiscernibility relation (i.e., if elementary sets of both relations are identical), then any attribute that belongs to the superset and not to the set is redundant. In the example from Table 1, let the set of attributes be the set {*Headache, Temperature*} and its superset be the set of all three attributes, i.e., the set {*Headache, Muscle_pain, Temperature*}. Elementary sets of the indiscernibility relation defined by the set {*Headache, Temperature*} are singletons, i.e., sets {e1}, {e2}, {e3}, {e4}, {e5}, and {e6}, and so are elementary sets of the indiscernibility relation defined by the set of all three attributes. Thus, the attribute *Muscle_pain* is redundant. On the other hand, the set {*Headache, Temperature*} does not contain any redundant attribute, since elementary sets for attribute sets {*Headache*} and {*Temperature*} are not singletons. Such a set of attributes, with no redundant attribute, is called *minimal* (or independent). The set *P* of attributes is the *reduct* (or covering) of another set *Q* of attributes if *P* is minimal and the indiscernibility relations, defined by *P* and *Q,* are the same (the last condition says that elementary sets, determined by indiscernibility relations defined by *P* and *Q,* are identical).

In our example, the set {*Headache, Temperature*} is a reduct of the original set of attributes {*Headache, Muscle_pain, Temperature*}. Table 2 presents a new information table based on this reduct.

So far we have not included a decision in our discussion. By analogy with attributes, we can define elementary sets associated with the decision as subsets of the set of all examples with the same value of the decision. Such subsets will be called concepts. For Tables 1 and 2, the concepts are {e1, e4, e5} and {e2, e3, e6}. The first concept corresponds to the set of all patients free from flu, the second one to the set of all patients sick with flu. The question is whether we may tell who is free from flu and who is sick with flu on the basis of the values of attributes in Table 2. To answer this question, we may observe that in terms of rough set theory, decision *Flu* depends on attributes *Headache* and *Temperature,* since all elementary sets of indiscernibility relation associated with {*Headache, Temperature*} are subsets of some concepts. As a matter of fact, one may induce the following rules from Table 2:

(Temperature, normal) -> (Flu, no),
(Headache, no) and (Temperature, high) -> (Flu,

no),
(Headache, yes) and (Temperature, high) -> (Flu, yes),
(Temperature, very_high) -> (Flu, yes).

Now, say that data from Table 2 is enhanced by two additional examples, e7 and e8, as presented in Table 3. Elementary sets of indiscernibility relation defined by attributes *Headache* and *Temperature* are {e1}, {e2}, {e3}, {e4}, {e5, e7}, and {e6, e8}, while concepts defined by decision *Flu* are {e1, e4, e5, e8} and {e2, e3, e6, e7}.

Obviously, in Table 3 the decision *Flu* does not depend on attributes *Headache* and *Temperature* since neither {e5, e7} nor {e6, e8} are subsets of any concept. In other words, neither concept is definable by the attribute set {*Headache, Temperature*}. We say that Table 3 is *inconsistent* because examples e5 and e7 are conflicting (or are inconsistent)—for both examples the value of any attribute is the same, yet the decision value is different. (Examples e6 and e8 are also conflicting.)

In this situation, rough set theory offers a tool to deal with inconsistencies. The idea is very simple—for each concept *X* the greatest definable set contained in *X* and the least definable set containing *X* are computed. The former set is called a *lower approximation* of *X*; the latter is called an *upper approximation* of *X*. In the case of Table 3, for the concept {e2, e3, e6, e7}, describing people sick with flu, the lower approximation is equal to the set {e2, e3}, and the upper approximation is equal to the set {e2, e3, e5, e6, e7, e8}, as depicted in Figure 1.

Similarly, for the concept {e2, e3, e6, e7}, the lower approximation is {e2, e3} and the upper approximation is {e2, e3, e5, e6, e7, e8}. Either of these two concepts is an example of a *rough set*, a set that is undefinable by given attributes. The set {e5, e6, e7, e8}, containing elements from the upper approximation of *X* that are not members of the lower approximation of *X*, is called a *boundary region*. Elements of the boundary region cannot be classified as members of the set *X*. On the other hand, rough sets may also be defined as sets having nonempty boundary regions.

For any concept, rules induced from its lower approximation are certainly valid (hence such rules are called *certain*). Rules induced from the upper approximation of the concept are possibly valid (and are called *possible*). For Table 3, certain rules are:

(Temperature, normal) -> (Flu, no),
(Headache, yes) and (Temperature, high) -> (Flu, yes),
(Headache, yes) and (Temperature, very_high) -> (Flu, yes);

**Table 2.** Reduced Information Table.

| | Attributes | | Decision |
|---|---|---|---|
| | Headache | Temperature | Flu |
| e1 | yes | normal | no |
| e2 | yes | high | yes |
| e3 | yes | very_high | yes |
| e4 | no | normal | no |
| e5 | no | high | no |
| e6 | no | very_high | yes |

**Table 3.** Inconsistent Information Table.

| | Attributes | | Decision |
|---|---|---|---|
| | Headache | Temperature | Flu |
| e1 | yes | normal | no |
| e2 | yes | high | yes |
| e3 | yes | very_high | yes |
| e4 | no | normal | no |
| e5 | no | high | no |
| e6 | no | very_high | yes |
| e7 | no | high | yes |
| e8 | no | very_high | no |

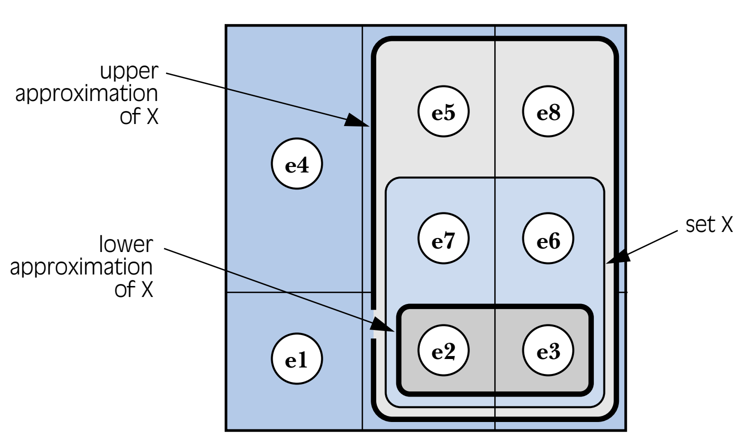


**Figure 1.** Lower and upper approximations of set X

and possible rules are:

(Headache, no) -> (Flu, no),
(Temperature, normal) -> (Flu, no),
(Temperature, high) -> (Flu, yes),
(Temperature, very_high) -> (Flu, yes).

A few measures of uncertainty were developed within rough set theory. The most frequently used are: a quality of lower approximation and a quality of upper approximation. For a given set $X$ of examples, not necessarily definable by a set $P$ of attributes, the quality of lower approximation is the ratio of the number of all elements in the lower approximation of $X$ to the total number of examples. Similarly, the quality of upper approximation is the ratio of the number of all elements in the upper approximation of $X$ to the total number of examples. Thus, in the example from Table 3, for the concept $X = \{e1, e4, e5, e8\}$, the quality of lower approximation is 0.25 and the quality of upper approximation is 0.75.

The quality of lower approximation may be interpreted as the ratio of the number of all certain classified examples by attributes from $P$ as being in $X$ to the number of all examples of the information table. It is a kind of relative frequency. Furthermore, the quality of lower approximation is a *belief function* according to Dempster-Shafer theory. Also, the quality of upper approximation is the ratio of the number of all possibly classified examples by attributes from $P$ as being in $X$ to the number of all examples of the system. Therefore, it is again a kind of relative frequency. The quality of upper approximation is a *plausibility function* from the Dempster-Shafer theory viewpoint [3]. Rough set theory is objective—for a given information table, qualities of corresponding approximations are computed. On the other hand, the Dempster-Shafer theory is subjective—it is assumed that values of belief (or plausibility) are given by an expert. (For further information about relations between rough set theory and the Dempster-Shafer theory, consult [15].)

## Applications of Rough Sets

The rough set theory has proved to be very useful in practice, as is clear from the record of many real-life applications. Descriptions of some of the implementations may be found in [9], [18], and [24]. However, most applications of rough set methodology are not covered by publications and are still in progress because of tedious collection of expermental data and development of new software.

The main problems that can be approached using rough set theory include data reduction (i.e., elimination of superfluous data), discovery of data dependencies, estimation of data significance, generation of decision (control) algorithms from data, approximate classification of data, discovery of similarities or differences in data, discovery of patterns in data, and discovery of cause-effect relationships.

In particular, the rough set approach has found interesting applications in medicine, pharmacology, business, banking, market research, engineering design, meteorology, vibration analysis, switching functions, conflict analysis, image processing, voice recognition, concurrent system analysis, decision analysis, character recognition, and other fields.

## Rough Sets, Knowledge Acquisition, and Machine Learning

Knowledge in the form of rules, induced by learning from training examples, may be used in rule-based expert systems. These rules are more general than information contained in the original input data, since new examples, which do not match examples from the original data, may be correctly classified by the rules.

The empirical learning system called Learning from Examples based on Rough Sets (LERS ), developed at the University of Kansas, consists of two options of machine learning from examples and two options of knowledge acquisition [3, 4]. Machine learning options produce a sufficient set of rules to cover all examples in the information table. Knowledge acquisition options produce much bigger sets of all rules that can be induced by a given option from the input data given by an information table. As was shown in [5], the machine learning approach is not adequate as a tool for knowledge acquisition when an expert system must deal with incomplete information. The knowledge acquisition options of the system LERS are examples of appropriate rule induction methods for building knowledge bases for expert systems working with incomplete information.

LERS may induce a set of rules from examples given in the form of an information table and may classify new examples using that set of rules. First LERS tests the input data for consistency. If data is inconsistent then lower and upper approximations of each concept are computed [4]. Now the user is offered an option to choose between two machine learning options and two knowledge acquisition options. If a machine learning option is used, then the system induces a single minimal discriminant description for each concept. If a knowledge acquisition option is applied, a complete set of rules is induced.

System LERS has been used for two years by NASA's Johnson Space Center as a tool to develop expert systems of the type most likely to be used in medical decision-making on board the space station *Freedom.*

Another application of LERS was enhancing facility compliance under Sections 311, 312, and 313 of Title III, the Emergency Planning and Community Right to Know. The project was funded by the U.S. Environmental Protection Agency.

LERS was also used in two medical applications, to compare the effects of warming devices for postoperative patients and to assess preterm labor risk for pregnant women. Prediction of preterm birth is a poorly understood domain. The existing manual methods of assessment of preterm birth are 17%–

38% accurate. The machine learning system was used for three different datasets about pregnant women. Rules induced by LERS were used in conjunction with a classification scheme of LERS, based on a "bucket brigade algorithm" of genetic algorithms and enhanced by partial matching. The resulting prediction of preterm birth in new, unseen cases is much more accurate (68%–90%).

Yet another interesting use of LERS was made in a study of global climate change. Rules, describing influence on global temperature, were induced from data characterized by attributes such as solar energy output, volcanic activity, Southern Oscillation Index, $CO_2$ trend, and $CO_2$ residual. Experts in the area gained new insight into the mechanism of global climate change, as reported in [6].

*Rough set theory offers effective methods that are applicable in many branches of AI.* **One of the advantages is that programs implementing its methods may easily run on parallel computers.**

## Rough Sets and Decision Analysis

The rough-set approach to decision analysis has been implemented in computer systems called RoughDAS and RoughClass, developed at the Poznan University of Technology in Poland. They perform the explanation and prescription tasks, respectively. The systems have been used in several domains of practical applications. Many of them have been presented in [18].

One application in medicine concerned verifications of indications for treatment of duodenal ulcer by highly selective vagotomy (HSV). Using rough set manipulations on a set of 122 patients described by a set of 11 preoperative attributes, the description has been reduced to five relevant attributes that ensure an acceptable quality of classification. The reduced attributes were based on tests that could show negative side-effects affecting patients. Application of 44 decision rules obtained from lower approximations of the classes of good and bad results of the operation to 70 new patients gave an increase of good results of HSV from 82% to 93% [17].

Another application concerned analysis of the relationship between the chemical structure and antimicrobial activity of 201 quaternary imidazolium compounds. The compounds were described by eight attributes concerning structure and were divided into five classes of activity. The reduct of attributes discovered with the RoughDAS system was composed of four attributes. Two sets of decision rules, one of 22 and one of 35 rules, gave clear advice on how to design new antimicrobially active compounds, in terms of the relevant characteristics of the structure [8].

In the domain of technical diagnostics, the rough set theory has been applied to analysis of diagnostic capacity of vibroacoustic symptoms. The rough set approach appeared to be a good tool for objective comparison of different methods of defining symptom limit values for both noise and vibration attributes used in the diagnostics of ball bearings. As a result of this study, the superiority of vibration symptoms over noise symptoms was established and a classifier of the technical state of rolling bearings has been built, consisting of 14 decision rules using three relevant symptoms (out of 12) [10].

In the domain of finance, the rough set approach has been used to identify firms with a bankruptcy risk. A real experience of Greek Industrial Development Bank ETEVA has been analyzed in order to assess its policy of granting credit to firms. The conclusions drawn from this analysis, expressed in clear terms of rules well supported by examples, were appreciated by financial experts. Both qualitative (unordered) and quantitative attributes were taken into account—the difficult task within the traditional multicriteria decision-making approach to construction of a value function. The sets of decision rules obtained use only 5%–7% of conditions from the initial table [19].

An interesting empirical study, now in progress, concerns the use of the rough set approach to reduction of data for a neural network classifying microscopic pictures of brain tumors. It has found that learning time accelerates data reduction by up to 4.72 times. A side result of this study is a claim that the minimum number of neurons in the hidden layer is equal to the cardinality of the smallest reduct. These promising results show that the rough set approach is a useful tool for preprocessing of data for neural networks.

## Rough Sets and Knowledge Discovery

As indicated previously, the applications of rough sets methodology actually implemented cover a wide spectrum of domains. An important application, that is enjoying increasing attention, is Knowledge Discovery in Databases (KDD). Knowledge discovery, or database mining, is a relatively new subdomain of AI concerned with the problem of digging out an extra dimension of nontrivial knowledge from ever-growing databases of corporate and other information. One of the primary tasks in this context is the discovery and characterization of inter-data connections or relationships; for example, between symptoms and diseases in medical databases. The discovered descriptions of fundamental factors occurring in such relationships help users better understand the nature of the phenomena about which the data is collected, or can be used for prediction [20, 21, 24, 25]. Another aspect, also handled using the rough set approach, is the discovery of abnormal patterns or behaviors in data for the purpose of detecting fraud or intrusion [23].

The methodologies for KDD are mostly rooted in prior research in statistics, database theory and machine learning [22]. However, newer developments in the area of rough sets position this methodology as one of the mainstream approaches to KDD problems [23–25].

The techniques of rough sets have been used for the purpose of KDD-related research for the last five years. In particular, the availability of PC-based commercial software systems for database mining, such as Datalogic [20], have made this technology accessible to users from different sectors of industry and science. Currently, the rough set methodology is being used, among other areas, in market research [25], medical data analysis [17], drug research [8], sensor data analysis for the purpose of control, and research leading to the design of new composite materials. The analysis of stock market data has confirmed some well-known market rules and has led to the discovery of some interesting new rules [25]. The knowledge discovery methodology that uses an extension of the original model of rough sets, called variable-precision rough sets, and the decision matrix method [23] have been implemented at the University of Regina in the newest set of workstation-based tools for knowledge discovery, called KDD-R. KDD-R was used for analysis of medical data and is currently supporting market research for the telecommunications industry.

## Conclusions

The rough set methodology has proved its soundness and usefulness in many real-life applications. Rough set theory offers effective methods that are applicable in many branches of AI. One of the advantages of rough set theory is that programs implementing its methods may easily run on parallel computers.

Nevertheless, several problems remain to be solved. Though the rough set theory has been developed on solid mathematical foundations, many theoretical problems still await proper clarification. Rough logic—a logic for imprecise reasoning based on rough set philosophy—seems to be the most important topic. Development of methods based on rough set theory for neural networks and genetic algorithms also seems to be very important. Rough controllers, i.e., controllers based on rough set theory, also seem to be a very promising area of applications. However, a qualitative control theory based on the rough set philosophy, must be created. The relationships of the rough set theory to nonstandard analysis, nonparametric statistics, and qualitative physics are other important topics.


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